The Right Math at the Right Time:
Addressing Mathematics Challenges Facing Michigan Colleges and Universities

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White Paper on RM@RT Strategy 1:
Align Learning Outcomes in Mathematics Pathway Courses

In early 2016, recognizing that successful implementation of the mathematics recommendation in the Michigan Transfer Agreement (MTA) would be critical to improving statewide transfer and credential completion, the Michigan Community College Association and the Michigan Association of State Universities established The Right Math at the Right Time (RM@RT) Task Force to strengthen the implementation of math pathways across Michigan's two- and four-year postsecondary institutions.

The Task Force members represented community colleges and universities, the Michigan Department of Education, and state mathematics associations. They were primarily recruited from the previous MTA Math Pathways Task Force or appointed by their institutions. The RM@RT Task Force began meeting in February 2016 and completed its initial work later that year with the recommendations in The Right Math at the Right Time: Addressing Mathematics Challenges Facing Michigan Colleges and Universities. That report outlined a structure to help Michigan colleges and universities review and revise mathematics curricula to meet the needs of students and employers, design new, high-quality mathematics learning experiences, align learning outcomes for developmental and gateway mathematics courses, and enable students who transfer to apply their math credits to their program of study. Today, the RM@RT Standing Committee continues this work under the umbrella of the Michigan Transfer Steering Committee, which oversees state-level transfer initiatives. The RM@RT Standing Committee is broadly charged with strengthening the implementation of math pathways across Michigan in accordance with the strategies listed in the 2016 recommendations. This document recommends learning outcomes and skill levels for the initial college-level courses which meet MTA requirements in Quantitative Reasoning, Statistics and College Algebra (preparation for Calculus).

Strategy 1: Establish a process to align learning outcomes for a set of introductory college level mathematics pathway courses across institutions and sectors.

Discussion: In 2014, the MTA Math Task Force defined the broad outlines of three mathematics pathways, including quantitative reasoning, statistics, and college algebra/preparation for calculus. These outlines, based on research and emerging practices in other states, include statements that address prerequisites, topics and descriptors for each pathway.

In 2016 the TSC charged the RM@RT standing committee to take the next steps to define actionable learning outcomes that will ensure the course-to-course transferability of a set of initial courses in the quantitative reasoning, statistics and calculus preparatory pathways. The standing committee convened three working groups focused on Strategy 1. Each working group included representatives from public community colleges and universities, and in some cases, independent colleges. These groups began work in early 2017. This document describes the goals and process for each working group as well as recommended skill levels and learning outcomes for the entry-level, college level courses in Quantitative Reasoning, Statistics and Preparation for Calculus.

Working Group Recommendations

1. Preparation for Calculus Working Group

Goal: The 2014 MTA Math Requirements define a "College Algebra Pathway—for students proceeding to programs that specifically require background in algebra, particularly STEM programs." Since most of these students will be required to enroll in calculus, the Preparation for Calculus working group developed outcomes for a precalculus course. Courses adopting these outcomes will transfer with course-to-course equivalence or substitute for a course also adopting these outcomes in a program of study (in cases where number of credits is an issue). These recommendations describe a **precalculus** course, but the working group recognizes that students may also qualify for calculus by completing a **college algebra** course **and** a **trigonometry** course that together cover the recommended outcomes.

Process: Members of the Preparation for Calculus working group met at Michigan State University on June 28, 2017 and began discussion in person. The focus was determining precalculus outcomes needed to prepare students for mainstream Calculus I/II. After this meeting the group researched

- Recommendations for college algebra from the MTA Math Task Force
- Mathematical Association of America (MAA) recommendations for precalculus
- Common requirements for precalculus at a variety of institutions

Recommendation: The **precalculus** course would qualify a student to take a calculus course and should be one that a mathematically **advanced** student would start with after completing high school algebra I and II. Students who have taken AP calculus in high school may use this course to remediate their algebra and trigonometric skills in preparation for college level calculus. This course should provide a strong foundation for critical mathematical thinking, be significantly beyond intermediate algebra, and include aspects of trigonometry.

The **college algebra** course should be one that a mathematically **capable** student would start with after completing high school algebra I and II. This course should provide a strong foundation for critical mathematical thinking, and be significantly beyond intermediate algebra. For students proceeding to calculus, the course should be followed by a course in **trigonometry.**

Skill Level: To be successful in a college algebra or precalculus course, students will be expected to apply advanced algebra skills similar to those taught in high school algebra II in the Michigan Merit Curriculum or in intermediate algebra. Institutions should establish their own guidelines for determining if a student can succeed in a one-semester precalculus course or should be placed into a two-course college algebra/trigonometry sequence to prepare for calculus.

Preparation for Calculus Outcomes:

The MAA Committee on the Undergraduate Program Mathematics (CUPM) established these suggested course objectives for college algebra and trigonometry in its 2015 report.

- Demonstrate knowledge of functions, including absolute values, polynomials (including polynomials of degree greater than 2 and the fundamental theorem of algebra), rational functions, logarithms, exponential functions, and inverse functions.
- Apply algebraic techniques in solving linear, quadratic, logarithmic, and exponential equations.
- Analyze equations of circles and properties of circles such as angle measure in both radians and degrees.
- Evaluate cosine, sine, and tangent for common angles (in all quadrants).
- Sketch trigonometric functions and state their domains.
- Recall and apply basic trigonometric identities such as the double angle, half-angle, and addition formulas.
- Graph functions by transformation rather than by plotting points.
- Topics such as sequences and series may also be included.

2. Introductory Statistics

Goal: The 2014 MTA Math Requirements define a “Statistics Pathway—for students proceeding to programs in business or social sciences.” The goal for the Statistics group was to define a set of course outcomes that not only satisfy the MTA requirements for general education but would also allow a course-to-course transfer as specific statistics courses across colleges and universities in Michigan.

Process: The Statistics Working Group had its first meeting on March 03, 2017 at Michigan State University. The meeting was followed by discussions via email. Another meeting was held at Michigan State University on June 28, 2017 as a part of the Michigan Mathematics Pathways Summit supported by the Dana Center. A final draft of the outcomes emerged after the meeting and more discussions via electronic means followed as more colleges and universities came on board.

Recommendation: Given the commonality of the topics covered in an introductory statistics course across colleges and universities in Michigan, our students are better served if the outcomes are not too broad but not too specific either. In a typical introduction to statistics course, topics are usually confined to data collection, descriptive statistics, probabilities, correlation and regression, and statistical inference.

It is strongly recommended that technology (graphing utilities, statistical software, or web-based statistical applications) is used to minimize tedious computations. To stress conceptual understanding and focus on statistical thinking, the proposed outcomes follow the recommendations by the American Statistical Association known as the Guidelines for Assessment and Instruction in Statistical Education (GAISE) which are strongly recommended when teaching and developing an introductory statistics course.

The MTA Agreement Handbook allows for courses that are equivalent to Introductory Statistics to satisfy the MTA math requirement if the course is organized or taught in the Math or Statistics Department or is cross-listed as a math or statistics course.

The following is a list of possible student learning outcomes for an introductory statistics course with a typical range of 3 to 4 semester hours. To qualify as a transferrable, college-level introductory statistics course in the State of Michigan the course should include 80% or more of the essential outcomes. The course may also include any or all optional outcomes.

Skill Level: To be successful in an introductory statistics course, students will be expected to apply basic algebra skills similar to those taught in high school algebra I in the Michigan Merit Curriculum or in beginning algebra.

Introductory Statistics Essential Outcomes:

1. Demonstrate understanding of the basic principles of data collection, observational study, and experimental design. This may include (but is not limited to) topics such as randomness, sampling error, sampling techniques, bias, blinding, and types of data.

2. Construct and interpret graphical and tabular displays of univariate data. These displays may include (but are not limited to): frequency distributions, pie charts, boxplots, stem plots, histograms.

3. Summarize distributions of univariate data using measures of central tendency, measures of dispersion, and measures of location.

4. Compare multiple data sets with graphical displays and numerical measures.

5. Perform basic probability computations. These may include (but are not limited to): the addition rule, the multiplication rule for independent events, and the complement rule.

6. Solve problems by applying appropriate probability distributions, which may include (but are not limited to) discrete, binomial, and normal probability distributions.

7. Use the Central Limit Theorem to model sampling distributions and compute probabilities based on sampling distributions.

8. Analyze bivariate quantitative data. This includes (but is not limited to), generating and interpreting r- and, r^2-values, scatterplots, and the least- squares regression lines for bivariate data.

9. Construct and interpret confidence intervals of proportion or mean for one population.

10. Construct and interpret confidence intervals for the difference of proportions or means for two populations.

11. Perform hypothesis tests for the means and proportions for one population. This includes interpreting p-value, type I and type II errors, and statistical and practical significance.

12. Perform hypothesis tests for the difference of proportions or means for two populations. This includes interpreting p-value, type I and type II errors, and statistical and practical significance.

13. Interpret and apply output from a statistical software package and/or a graphing utility.

14. Interpret and apply appropriate statistical techniques and concepts to real-life data and situations in order to make decisions and/or draw conclusions.

Introductory Statistics Optional Outcomes:

15. Perform intermediate probability computations. These may include (but are not limited to): the multiplication rule for dependent events, conditional probability, and Bayes Theorem.

16. Analyze bivariate qualitative data presented in two-way tables and interpret relationships between categorical variables. This may include (but is not limited to) computing probabilities, identifying lurking variables, explaining Simpson’s Paradox, and conducting appropriate chi-square tests.

17. Perform more advanced hypothesis tests such as the goodness-of-fit test, independence test, and ANOVA.

3. Quantitative Reasoning

Goal: The 2014 MTA Math requirements define a "Quantitative Reasoning Pathway—for students proceeding to majors not requiring statistics or calculus." The goal of the Quantitative Reasoning working group was to define a set of course outcomes that are broad enough to encompass the variety of quantitative reasoning courses as they exist today, yet specific enough that colleges and universities will accept course-to-course transfers for courses that satisfy the outcomes. These recommendations will bolster the quantitative reasoning portion of the MTA agreement so that courses taken at other institutions not only satisfy general education requirements but also transfer as specific mathematics courses statewide.

Process: The QR Working Group began with a meeting February 3 at Michigan State University and continued with online discussion of the results of that meeting. There was a follow up meeting at the MSU Stem Pathway Meeting hosted in part by the Dana Center on June 28, 2017. More online discussion followed. Read more about the process and supporting documents at http://bit.ly/RMARTQR

Recommendation: The recommended outcomes come from The Association of American Colleges and Universities Quantitative Literacy Rubric. The Assumptions outcome was removed because it was thought that it is a natural part of Representation and not a stand-alone outcome.

Skill Level: To be successful in a quantitative reasoning course, students will be expected to apply basic algebra skills similar to those taught in high school algebra I in the Michigan Merit Curriculum or in beginning algebra.

Quantitative Reasoning Outcomes:

- **Interpretation:** Ability to explain information presented in mathematical forms (e.g., equations, graphs, diagrams, tables, words)
- **Representation:** Ability to convert relevant information into various mathematical forms (e.g., equations, graphs, diagrams, tables, words)
- **Calculation:** Ability to identify and perform appropriate calculations and communicate results
- **Application / Analysis:** Ability to make judgments and draw appropriate conclusions based on the quantitative analysis of data, while recognizing the limits of this analysis
- **Communication:** Expressing quantitative evidence in support of the argument or purpose of the work (in terms of what evidence is used and how it is formatted, presented, and contextualized

Guidance for Implementation:

The RM@ RT Standing Committee recognizes these recommendations for learning outcomes and skill levels for the initial college-level courses in the three MTA mathematics pathways—quantitative reasoning, introductory statistics and preparation for calculus. Students who successfully complete these courses should meet the MTA mathematics requirement and either receive direct credit for an equivalent course offered at a transfer institution or be permitted to enroll in the next course in the sequence to which the course is a prerequisite. To act on the recommendations, the committee suggests that institutions wishing to participate in this agreement:

1. *Adopt* the statewide learning outcomes in their own entry-level college-level courses in each pathway offered at the institution. Mathematics departments who determine that their courses meet at least 80% of the recommended outcomes may wish to list the outcomes met on the course syllabus.

2. *Accept* entry-level, college-level courses in each pathway for transfer with course-to-course equivalence if the outcomes are met. Institutions should consider reviewing their policies regarding acceptance of transfer credit to reduce awards of general, elective and departmental credit for initial college-level courses that meet the MTA mathematics requirement.

3. *Apply* entry-level college-level courses adopting the statewide learning outcome for each pathway to programs of study as specified by program requirements, either through course-to-course transfer or by course substitution. To extend the application of MTA mathematics courses beyond general education requirements, programs of study should consider specifying a mathematics pathway that is most relevant to their discipline.

The RM@RT Standing Committee recommends that institutions use the new Michigan Transfer Network (MTN) website, scheduled to come online in 2019, to demonstrate their compliance with these recommendations. The new MTN will be a repository for transfer information for both students and institutions. While the current MTN provides only course-to-course equivalency between public and independent colleges and universities, the new MTN will provide expanded information about course-to-course equivalency, how to complete the MTA, and comprehensive details about associate to bachelor's degree transfer pathways in selected disciplines. The RM@RT Standing Committee will use the MTN to track statewide progress in adopting, accepting and applying mathematics courses with the recommended learning outcomes.

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